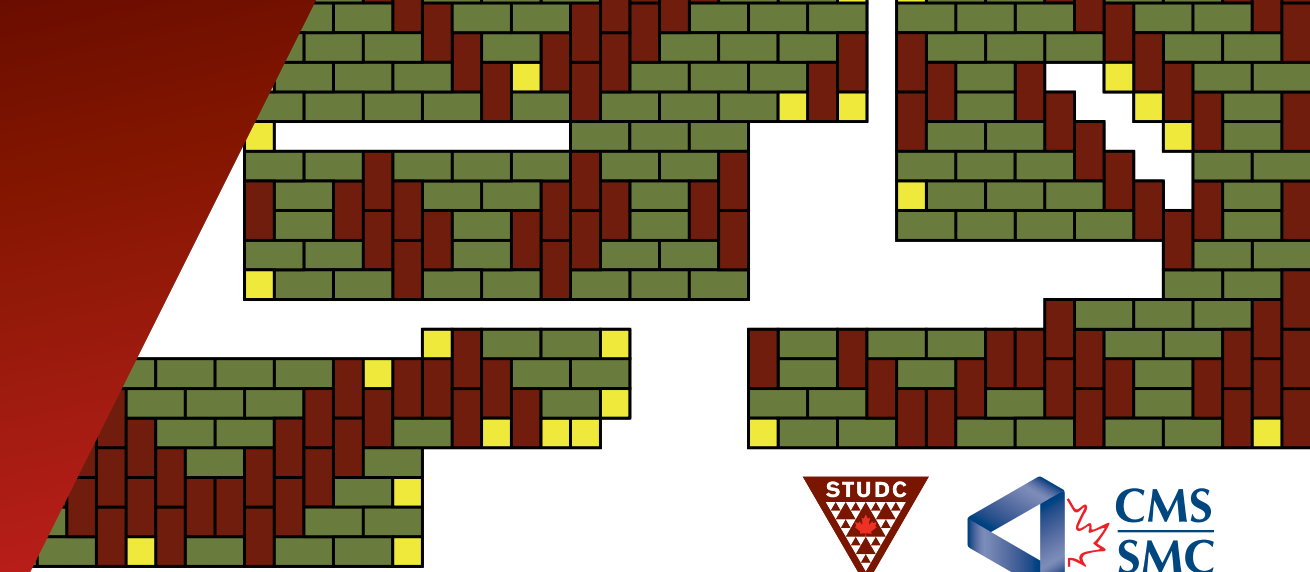


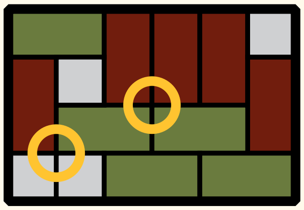
Notes from the Margin

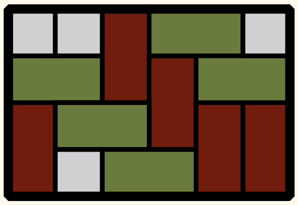
Volume II • 2011

Japanese tatami mat tilings: No four tiles meet

By Alejandro Erickson

Tatami mats are the type of mats used in traditional Japanese rooms. Often, an arrangement in which four mats meet at a point is considered unlucky, perhaps because the word "four" sounds like the word "death" in Japanese. So, a "lucky" layout has no "+" shapes formed by the lines where mats meet. Compare the following two arrangements.





Traditionally made of rice and rush straw, tatami are made in uniform sizes. A full mat measures 3′×6′, which we represent by a 1×2 tile and call it a *dimer* ; a half mat measures 3′×3′, corresponds to a 1×1 tile and is called a *monomer*. We tip our hat to statistical physics, to which we owe much credit for the popularity of classical dimer tilings[1]. Perhaps the best frame of reference, however, is that of recreational mathematics. The simple tatami condition, that no four tiles meet, and the resulting patterns are appealing for their own sake.

Surprisingly, a monomer-dimer tatami tiling of a grid is formed by only four distinct structural features. The tiling in *Figure 1* is an example containing all four types. Compare this with *Figure 2*, where these features are highlighted. What is going on?

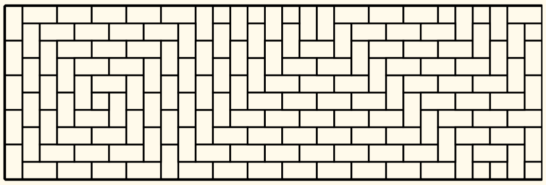


Figure 1: A tatami tiling of the 10×31 grid with 10 monomers, 64 vertical dimers and 86 horizontal dimers. Compare this with *Figure 2*.

Consider the situation where a horizontal dimer shares an edge with a vertical dimer: . Either the shared edge touches a boundary of the grid, or the placement of another dimer is forced by the tatami condition: . This creates a new shared edge ; the same logic applies , building outward until the boundary is reached. A bit of case analysis reveals more about this propagation, called a *ray*. The cases come by

[1] Richard Kenyon has written extensively about this and offers the information for free on his website, http://www.math.brown.edu/~rkenyon/.

Preamble

By Kseniya Garaschuk

Kseniya Garaschuk
University of Victoria

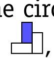
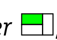
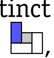
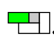
I'm constantly told that my volume is inverse proportional to my size. I eat apples with their cores and oranges with the peel. I'm 80's to the core: big curly hair, big sunglasses, high heels, and love for everything disco, ABBA and Freddy Mercury. No blue eyeshadow though - that's just ridiculous.

We, mathematicians, are all a little bit nuts. Among our ranks are coffee-addicts, low talkers, close talkers, and statisticians. Sometimes, joined by some common funky interest, we even combine together to form sets: think Bourbaki, the Pythagoreans and the Klein four group. And yet, even though each of us possesses some non-ordinary qualities, we love pointing out other mathematician's oddities: "Check out his crazy hair", "She constantly mutters to herself", "That guy's clothes are always monochromatic". Admit it — noticing a peculiar mathematician makes you feel better about yourself. It's comforting to know that there is always someone quirkier than you are.

If you need a boost of crazy today, look no further — this issue's contributors are really something else. We hear from Alejandro, who investigates auspicious patterns in Japanese rugs; David, who simply must solve an equation, but is stuck in a field where freshman dreams come true while quadratic formula fails; Louis qui faisait des maths tout en savourant une des meilleures bières que l'Europe a à offrir. And for those of you who take pride in being the odd one in the room, we have some more standard fare: administrative notices, a few shorter pieces and an all new distractions page.

To better acquaint you with the quirks of our contributing team, we've asked each of them some personal questions. You'll find their answers and their pictures next to their contribution.

As always, we welcome comments, suggestions and of course contributions to future editions of the Margin. E-mail me at student-editor@cms.math.ca with any kind of crazy notion that you would like to share.

considering the tile covering the grid-square at the beginning of the ray, indicated by the circle . If it is covered by a vertical dimer , it moves the start of the ray. Otherwise, we see one of the four distinct features: a *bidimer* , a *vee* , a *vortex* , and a *loner* . Each of these four features is the source of at least one ray in the tiling and all rays begin at either a bidimer, vee, vortex or loner. The different types of features are depicted in *Figure 3*. The sources reveal interesting structures of tatami tilings. For example, a bidimer or a vortex may appear anywhere in a tiling, as long as the coloured tiles are within its boundaries. The vees and loners, on the other hand, must appear along a boundary, as shown in *Figure 2*.

Let us now look at tatami tilings of the n×n grid which have a bidimer. Consider the (vertical) bidimer in *Figure 4*, and notice that the placement of the blue and green tiles is forced by the tatami condition. The corners remaining to be tiled are separated from each other by rays extending from the bidimer, regardless of where the bidimer appears. Any feature besides the loner sends rays in more than one direction, which would conflict with the existing bidimer. Thus, the only structural features that may appear there are loners.

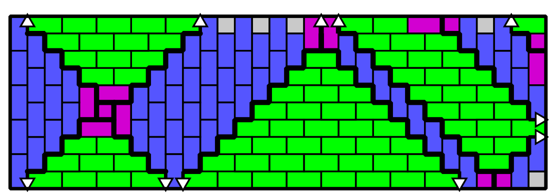


Figure 2: A tiling showing all four types of sources. Coloured in magenta, from left to right are a clockwise vortex, a vertical bidimer, a loner, a vee, and another loner.

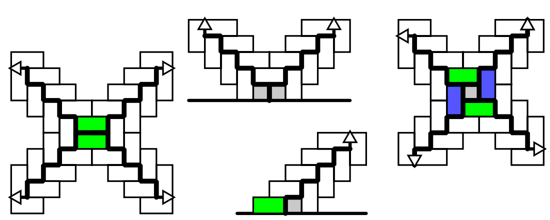


Figure 3: The four features of a tatami tiling, up to rotation and reflection from left to right: horizontal bidimer, vee, loner, counterclockwise vortex.

Adding or removing loners is equivalent to changing the orientations of tiles in diagonal strips, coloured magenta or grey in *Figure 4*. We call these *diagonals*, and they can either be in a flipped or unflipped state. As it turns out, the number of monomers in the tiling is decided by the minimum number of grid squares, k, between the center of the bidimer and the boundary. There is exactly one monomer in each diagonal, and one or two more of them elsewhere in the tiling, coloured in red in *Figure 4*. Omitting the details (see [1] for a complete proof), there are $(m + 1)2^{m+1}$ tilings with m monomers and a bidimer, when $m < n$. A similar argument reveals that the number of m-monomer tilings with vortices, where $m < n$, is $m2^m$.

Such round numbers compel us to find nice equivalence classes. Perhaps the tilings counted above can be divided into $m + 1$ classes of size 2^{m+1}? Consider the following two theorems.

Theorem 1. [1] If n and m have the same parity, and $m < n$, then the number of $n \times n$ tatami tilings with m monomers is $m2^m + (m + 1)2^{m+1}$.

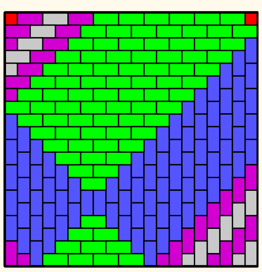


Figure 4: Vertical bidimer. The placement of green and blue tiles is forced by the bidimer. The remaining grey and magenta diagonals can be flipped independently.

Theorem 2. [2] The number of $n \times n$ tatami tilings is $n2^{n-1}$.

Combining the above and taking for granted that there are no tatami tilings with $m > n$, there are $2^{n-1}(3n-4) + 2$ tatami tilings of the $n \times n$ grid. Now, toss in a curious fact from The On-Line Encyclopedia of Integer Sequences [3].

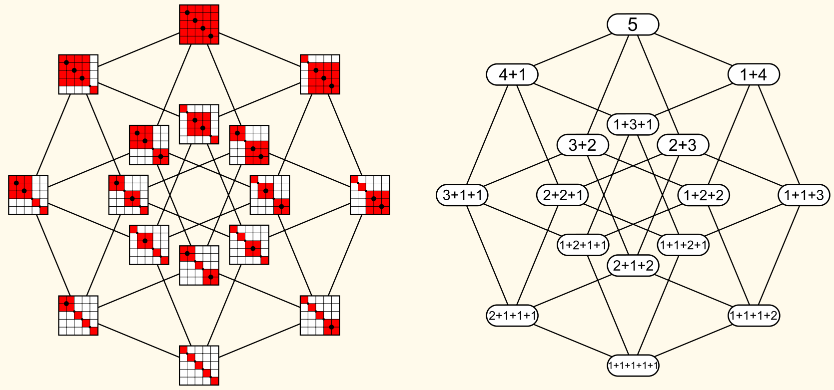


Figure 5: Is there a aesthetically pleasing mapping from $n \times n$ tatami tilings to red squares? Image credit: [4]

Fact. The sum of the squares of all parts of all compositions of n is $2^{n-1}(3n - 4) + 2$.

This last fact is represented by the number of red squares in the Hasse diagram of *Figure 5* for $n = 5$. It would be nice to find an appealing bijection between the tilings and the red squares. I invite you to arrange all $2^{n-1}(3n - 4) + 2$ tatami tilings of the $n \times n$ grid in an aesthetically and mathematically appealing way and write to **alejandro.erickson@gmail.com.**

Alejandro Erickson
University of Victoria

I've caught the geometry bug. This year I built a 1.5 storey icosahedron in my dad's front yard, and I have the materials to build another which will be 30 feet between opposing faces. I will need a larger yard though.

Note from the editor: He forgot to mention that his dad's house is bright neon green. Actually.

References

[1] Alejandro Erickson and Mark Schurch. Enumerating tatami mat arrangements of square grids. Proceedings of the International Workshop on Combinatorial Algorithms (IWOCA), Lecture Notes in Computer Science (LNCS). Springer Berlin / Heidelberg, 2011.

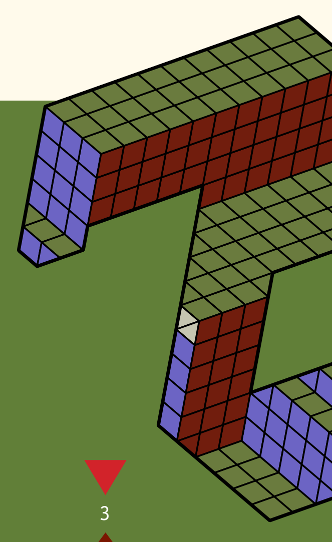
[2] Alejandro Erickson, Frank Ruskey, Mark Schurch, and Jennifer Woodcock. Monomer-dimer tatami tilings of rectangular regions. The Electronic Journal of Combinatorics, 18(1):24, 2011.

[3] Vladeta Jovovic. Comment on a027992. http://oeis.org/A027992, 2005.

[4] Tilman Piesk. Binary and compositions 5. http://commons.wikimedia.org/wiki/File:Binary_and_compositions_5.svg, 2010.

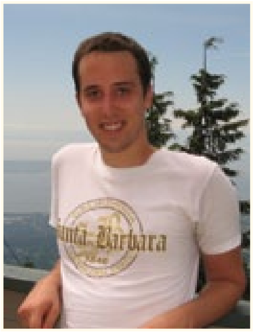
The 2011 CMS Winter Meeting will take place December 10 - 12, 2011 hosted by Ryerson and York Universities in Toronto. Studc is planning a number of exciting student events to be held during the meeting, including a poster session, a CV writing workshop and a social. Check the meeting website at http://cms.math.ca/Events/winter11/ for more information.

La Réunion d'hiver de 2011 de la SMC aura lieu du 10 au 12 décembre 2011 et est hébergés par les universités Ryerson et York, à Toronto. Studc prévoit un certain nombre de manifestations excitant pour les étudiants qui se tiendra lors de la réunion, notamment une séance d'affiches, un atelier de rédaction de CV et un événement social. Vérifiez le site Web de la Réunion à http://cms.math.ca/Reunions/hiver11/ pour plus amples informations.



S'échanger les idées

Par Louis-Xavier Proulx



Louis-Xavier Proulx
Université de Montréal

I have solved many problems (math or coding) while sleeping or just before falling asleep. The last one I can remember was to prove something in functional analysis by showing that a sequence was Cauchy to prove the compactness of an operator. Nothing so exciting... Remember that I'm an applied math guy...

Déjà une ou deux années de scolarité universitaire réussies et vous trouvez déjà votre vie un peu trop routinière à votre goût. Vous avez peut-être besoin de nouveaux défis, de changer d'air. Alors pourquoi ne pas aller étudier sur un campus étranger pour une session ou deux?

L'échange étudiant est une excellent façon d'aller voir ailleurs. Le film L'Auberge Espagnole a largement contribué à faire connaître les programmes d'échange et à les rendre populaires. Cherchant à mousser leur vocation internationale, les universités de ce monde ont conclu de nombreuses ententes ces dernières années pour s'échanger des étudiants. Ils offrent souvent des bourses ainsi qu'un support logistique pour passer au travers du long processus des demandes et de la préparation avant le départ. Il faut savoir qu'un échange étudiant se prépare presqu'un an à l'avance. Pour un départ en septembre ou en janvier, la candidature doit être déposée à l'hiver précédent.

J'ai eu l'opportunité d'étudier à l'Université Libre de Bruxelles en Belgique pour l'année scolaire 2006-2007. Plusieurs raisons que je ne détaillerai pas ici m'ont amené à faire ce choix. Chaque destination a son lot d'avantages et d'inconvénients. Des facteurs importants à considérer sont le coût de la vie, la disponibilité des logements, la langue parlée et même la météo! Il faut aussi vérifier les choix de cours offerts à l'université d'accueil, afin de s'assurer que vous pourrez trouver des équivalences avec les cours de votre programme d'études de votre établissement d'attache. Cette étape est importante, car les équivalences doivent être approuvées par votre directeur de programme, bien que souvent il faut revoir les choix de cours une fois arrivé à destination.

Mon échange n'a pas toujours été facile. La ciel en Belgique est plutôt gris et il tombe (trop) souvent une légère pluie. Je ne pensais pas que mon moral pouvait être affecté à ce point par la météo. Il faut aussi savoir que la bureaucratie belge partage beaucoup de similitudes avec la maison des fous des 12 travaux d'Astérix. Pour mon choix d'université, le programme de mathématiques mettait l'accent sur l'algèbre, ce qui est moins intéressant pour quelqu'un avec des intérêts en mathématiques appliquées. Par contre, les classes étaient petites, les étudiants pouvaient s'entendre sur un nouvel horaire pour le cours, les profs étaient près de leurs étudiants et très compréhensifs.

La Belgique est particulièrement réputée pour ses frites, son chocolat, ses gaufres et sa bière, une réputation entièrement fondée. La bière belge me manque énormément, vous ne pouvez même pas imaginer combien d'argent j'ai pu dépenser en bière durant cette année. La proximité de la Belgique à plusieurs autres pays m'a permis de faire plusieurs voyages au cours de la session puisque les congés abondent dans le calendrier scolaire belge. Les Belges font preuve d'une très grande créativité lorsqu'il est question de faire la fête, soit par les nombreux festivals au quatre coins du pays ou par les parties arrosés sur le campus. J'ai eu la chance de partager une grande maison de 10 chambres avec 9 autres étudiants étrangers, devenus de bons amis. Ce fût une année mémorable qui a malheureusement passé beaucoup trop vite.

Un échange étudiant est une expérience enrichissante qui surpasse largement tous ses moins bons côtés. Les étudiants du Québec pourront consulter la page de la CRÉPUQ de plus amples informations. Vous pouvez également passer au bureau responsable des échanges de votre université. La SMC et le CRSNG proposent une bourse pour aller étudier à l'Université indépendante de Moscou à l'hiver 2012. La date limite pour le dépôt de la demande est le 30 septembre.

References

http://echanges-etudiants.crepuq.qc.ca **et** http://cms.math.ca/Bourses/Moscou

The organizers of the **Fields Undergraduate Network (FUN) Discrete Mathematics meeting**, held at Carleton University on July 22, 2011, would like to sincerely thank the CMS Student Committee for their generous support of the FUN network.

The discrete mathematics event consisted of three excellent talks from Daniel Panario (Carleton), Mateja Sajna (Ottawa) and Aaron Williams (Carleton) on various areas of discrete mathematics. The talks were followed by a panel discussion, headed by the three speakers, with informative topics ranging from how to break into research as an undergraduate student to how quirky discrete mathematicians are relative to other mathematicians (the end result: we're all mathematicians and so we are probably all a little strange to begin with!).

Look forward to future collaborations between the Fields Undergraduate Network and the CMS Student Committee in the near future!

A pretty application of normal bases

By David Thomson

Special thanks to Dr. R.C. Mullin (Waterloo) for originally presenting me with this problem.

Let q be a power of a prime. It is well known that there is a unique finite field of order q, denoted $\mathbb{F}_q$. Extensions of finite fields can be viewed as vector spaces over the base field, and we often use the two notions interchangeably. For any finite field, we can identify bases of $\mathbb{F}_q^n$ with those of $\mathbb{F}_{q^n}$. In applications such as coding theory, cryptography and signal processing the most common basis used is a *normal basis*.

Let $\alpha \in \mathbb{F}_{q^n}$. If $N = \{\alpha, \alpha^q, \ldots, \alpha^{q^{n-1}}\}$ forms a linearly independent set over $\mathbb{F}_q$, then N is a normal basis and we call α a normal element of $\mathbb{F}_{q^n}$. Representations using normal bases are useful due to their eciency in exponentiation. First, recall that in a finite field we have the *Freshman's Dream*: $(a+b)^q = a^q + b^q$ (if only life were always so easy).

Then, if $\eta = a_0\alpha + a_1\alpha^q + \cdots + a_{n-1}\alpha\, q^{n-1}$, we have $\eta^q = a_0\alpha^q + a_1\alpha^{q^2} + \cdots + a_{n-1}\alpha$ since $\alpha^{q^n} = \alpha$ and $a_i^q = a_i$. In vector notation, we have $(a_0, a_1, \ldots, a_{n-1})^q = (a_{n-1}, a_0, \ldots, a_{n-2})$, and so exponentiation by q is simply a bit-shift. Using repeated-squaring, exponentiation to any m thus requires roughly $\log_q(m)$ multiplications.

In practical applications (I swear, this is *not* a tenuous link to reality!), the most interesting finite fields are extensions of $\mathbb{F}_2$. However, leaving the characteristic 0 world introduces problems. For example, suppose we want to solve the quadratic equation in $\mathbb{F}_{2^n}$. We do not have the quadratic formula in characteristic 2 (why?), so we must find an alternate solution.

Consider the quadratic equation $X^2 + AX + B = 0$, $AB \neq 0$, over $\mathbb{F}_{2^n}$. First, substitute $Y \leftarrow X/A$ and normalize by A^2 to obtain $Y^2 + Y + C = 0$. Let $N = \{\alpha, \alpha^2, \ldots, \alpha^{2^{n-1}}\}$ form a normal basis of $\mathbb{F}_{2^n}$ over $\mathbb{F}_2$. Suppose $(y_0, y_1, \ldots, y_{n-1})$ is a solution of $Y^2 + Y + C = 0$ and let $C = (c_0, c_1, \ldots, c_{n-1})$. Using a normal basis representation, we have $(y_{n-1}, y_0, \ldots, y_{n-2}) + (y_0, y_1, \ldots, y_{n-1}) = (c_0, c_1, \ldots, c_{n-1})$. Comparing entries gives the system of equations

$$\begin{array}{ccccc} y_{n-1} & + & y_0 & = & c_0 \\ y_0 & + & y_1 & = & c_1 \\ \vdots & & \vdots & & \vdots \\ y_{n-2} & + & y_{n-1} & = & c_{n-1} \end{array}$$

David Thomson
Carleton University

I've been told that I talk to complete strangers like they're long-lost friends. They can be a bartender or a random person at the bus stop, regardless, I make sure that everyone feels welcome and included. I once made a friend in Revelstoke BC waiting for a Greyhound to Vancouver, only to part with an awkward "See you... never? Take care." in Salmon Arm.

Adding all of the equations gives

$$2\sum_{i=0}^{n-1} y_i = 0 = c_0 + c_1 + \cdots c_{n-1} = \mathrm{Tr}_{2^n/2}(C),$$

where $\mathrm{Tr}_{2^n/2}$ denotes the $\mathbb{F}_2$-trace function. Therefore, the equation $Y^2 + Y + C = 0$ has a solution if and only if $\mathrm{Tr}_{2^n/2}(C) = 0$.

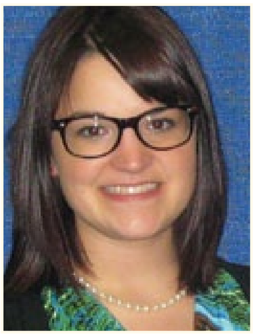
Furthermore, the two solutions are given by

$$\begin{aligned} y_0 &= \delta, \\ y_1 &= \delta + c_1, \\ y_2 &= \delta + c_1 + c_2, \\ &\vdots \\ y_{n-1} &= \delta + c_1 + \cdots + c_{n-1}, \end{aligned}$$

where $\delta = 0,1$.

Different learners, different methods

By Amber Church



Amber Church
University of Guelph
In the summer I work at a forest fire management centre as a communications officer, dispatching initial attack aircraft. I also have excellent taste in eyewear.

We're all familiar with that moment when understanding just clicks into place, but do you remember the way the concept was explained that enabled the understanding? Every individual student learns best in a particular way - be it a diagram, verbal explanation, step by step written process or a "real-life" explanation.

I'm sure that first year calculus isn't enthralling for all students precisely due to the very teacher-centric methods of instruction commonly used in university classes. So when given the opportunity to run a lab or a seminar, take a break from the lecture paradigm and use a student-directed approach. This can be achieved by employing a three-part lesson plan commonly used in high school classrooms. Part 1 is dedicated to "minds on" learning, presenting both a review of previously learned concepts and new ideas that will relate directly to the current day's material. Part 2 is the "action" stage, where students explore new mathematical concepts on their own using problem solving and reasoning to make connections. This stage can be time-consuming, but it is worthwhile as it leads to a much deeper understanding of the material. Finally, Part 3, "consolidate", allows time for class discussion, reflection and the debriefing of the new method or theory the students should have discovered themselves.

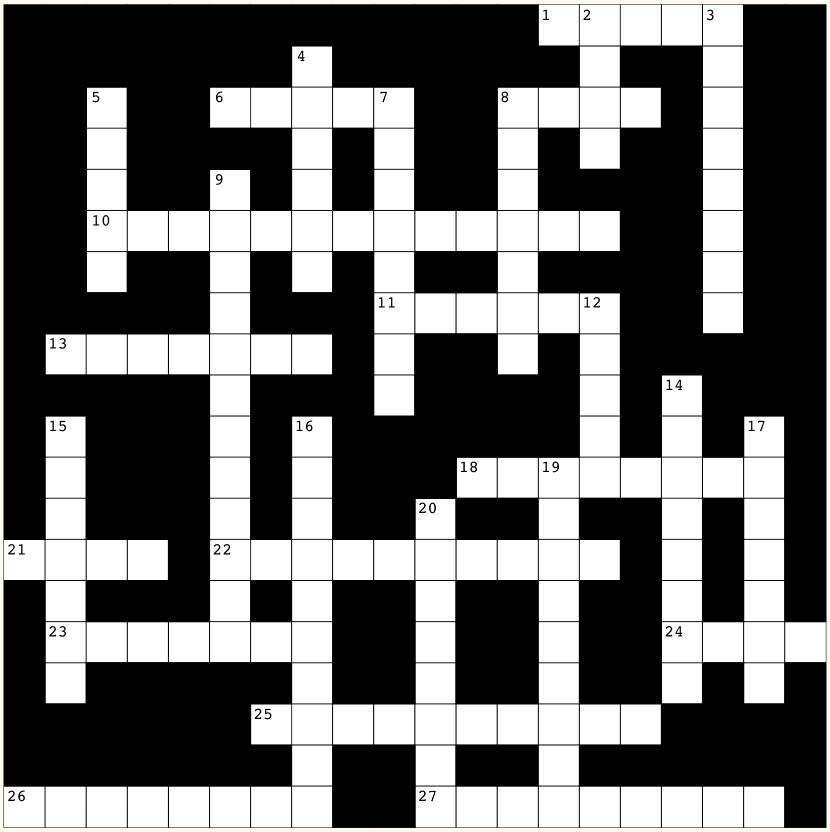
While the above is the reverse of the traditional method of presenting a concept with an example and then allowing for practice, this three-part lesson plan has the advantage of students taking an active part in their own learning. This, in turn, results in a more meaningful understanding of the material.

Participate in the **student poster sessions at the CMS Winter 2011 meeting** and compete for prizes! Enjoy presenting your research in a relaxed one-on-one atmosphere with experts in the field offering you immediate feedback. Check the conference web-site for tips that will help you create a winning poster.

Participer aux **séances d'affiches d'étudiants à la Réunion d'hiver 2011 de la SMC** et gagner des prix! Profitez de présenter votre recherche dans un atmosphère détendue en tête-à-une avec des experts dans le domaine, vous offrant commentaire immédiate. Vérifiez le site Web de la Réunion pour obtenir des conseils qui vous aidera à créer une affiche gagnante.

The Distractions Page

Life as a Graduate Student



Across

1 European version of mathematics
6 A rock band or an algebraic structure
8 Mathematician's social petrol
10 Complete
11 Sense of foreboding
13 Allocating scores
18 A distant scrutineer
21 Finite Fields bible: _____ and Niederreiter
22 Co-worker
23 An academic's direct progenitor
24 {}
25 Academic toddlers
26 Not continuous
27 T_2 Space

Down

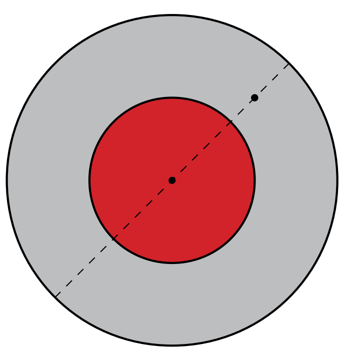
2 *ab* = *ba* namesake
3 A property of isosceles triangles
4 Mathematician's professional petrol
5 Where vectors live
7 Common response to "Name a graph"
8 A source in a Tatami tiling
9 A layman's common question
12 1954 Fields Winner
14 Of a single property or a feeling during *13. Across*
15 An internal place to talk
16 *15.* Down on a larger scale
17 Proof weasel word
19 Addenda to lectures
20 Explore

So what is the probability of hitting the bull's eye?

Let's say you're throwing darts at a very simple dart board: the radius of the whole dart board is 2, while the radius of the bull's eye is 1. What is the probability of hitting the bull's eye? Here are two possible reasonings and they surely can't both be correct, so find a problem.

With a random throw, the probability of hitting the bull's eye is the ratio of the area of bull's eye to the area of the entire dart board:

$$\frac{\pi(1)^2}{\pi(2)^2} = \frac{1}{4}$$



There is a unique line through the point where your dart lands and the center of the dart board. Half of this line lies inside the bull's eye and half of it lies outside of it, making the probability of hitting the bull's eye equal to 1/2

The 1/2 is incorrect. It is the probability of hitting bull's eye given that your dart lands on that particular line. However, the probability of hitting a given line is zero.

Chris F. Duffy
University of Victoria

The best place I've ever done math was lying on a piece of dried driftwood while listening to the waves gently lap the shore. A close second was while rocking a mean ukulele.

Studc in the Community

Here are some of the initiatives the Studc supported and organized in the past year:

- Launch of the Notes from the Margin, our semi-annual publication
- Student social at the 5th Annual meeting of the Prairie Network for Research in the Mathematical Sciences held at the University of Regina
- Indiscreet Math Night at the 2011 Ottawa-Carleton Discrete Math Days
- Student poster session and the Glow in the Dark Mini Putt during the CMS Summer meeting at the University of Alberta
- The 18th edition of the Canadian Undergraduate Mathematics Conference (CUMC) held at Université Laval
- CUMC Award for Excellence - congratulations to David Peterson of University of Northern British Columbia
- Fields Undergraduate Network Discrete Math meeting held at Carleton University

Errata

Patricia Hennesey from Carleton pointed out that the solution to Dr. Johnson's latex problem (Volume I, 2011) is not feasible. The assumption that both the inside and the outside of the surgical glove are sterile is incorrect. We apologise for any infection spread over the last month due to an application of the solution.

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