St. John's, Newfoundland & Labrador

Come East and Go Further

Please post and distribute to undergraduate students.

Department of Mathematics & Statistics
Memorial University
St. John's, NL, A1C 5S7
CANADA

www.mun.ca/math
math-head@mun.ca
(709) 864-8783 / 84

MEMORIAL UNIVERSITY

An Invitation to Canada's Mathematics and Statistics Undergraduates...

If you are hopeful of winning an **NSERC Undergraduate Student Award (USRA)** for research next summer, please consider visiting the Department of Mathematics and Statistics of Memorial University.

This is one of Canada's top universities and the largest in Atlantic Canada. With over 32 faculty members, half of whom have been hired since 2005, the Department of Mathematics and Statistics is quickly becoming one of the best. One fifth of our faculty hold the highest rank of university research professor.

Attached are several proposals prepared with undergraduate research in mind. If you're interested in coming east, if you're a Canadian citizen or have permanent resident status, and if you have an A average, please contact Dr. Edgar Goodaire at *math-head@mun.ca* before January 9, 2012.

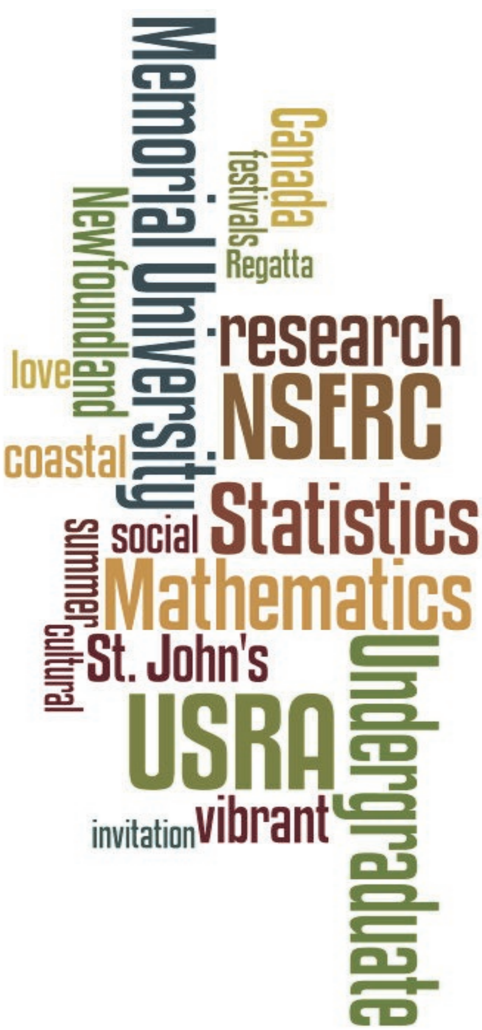
You will LOVE St. John's, the oldest city in North America, with a vibrant cultural and social life, great coastal walking trails, the scene of fabulous summer folk and rock festivals and home to a Regatta that dates to 1816.

Measurement error and genetic epidemiology of obesity in Newfoundland

Obesity is the most common metabolic condition in developed countries and is considered to be the second leading avoidable cause of mortality for adults in Western countries. Recent reports indicate that Newfoundland and Labrador has the second highest prevalence of overweight and obese adults in Canada. This complex trait is determined by multiple environmental factors that interact with one another in complicated ways. The existing studies examine such factors under the assumption that they are measured accurately. However, unobserved or error-prone environmental factors and/or genotyping errors are unavoidable. It is now well known that measurement error can influence the results of the study. The impact of ignoring these errors varies from bias and large variability in estimators to low power or even false-negative results in detecting genetic associations. In order to improve the accuracy and precision in the assessment of both genetic and environmental factors, one needs to take into account these errors. Our objective is to develop a general statistical framework able to efficiently characterize interactions between some environmental and genetic factors that link to the development of obesity, where both factors are prone to error. Our ultimate goal is to discover and characterize new biomarkers of abnormal growth, resulting in the development of new approaches to the diagnosis and treatment of obesity in the province.

This project benefits from a close collaboration with Professor Guang Sun at the Faculty of Medicine at Memorial University. He is an expert in the field of genetics and nutritional factors responsible for obesity and diabetes. We will apply our methodology to the genetics data provided by Dr. Sun's lab. This data includes some environmental factors as well as genotyping data of over 3000 individuals from the province. Summer students will be trained to apply different algorithms to estimate genotyping error rates on the data, and also to do some advance analysis and modeling.

Dr. Taraneh Abarin
Assistant Professor in Statistics
tabarin@mun.ca



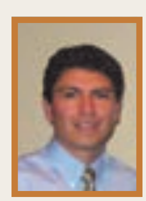
Curvature in group theory

Geometric group theory is the study of algebraic properties of a group by the use of geometry. Many questions in this area can be attacked by undergraduate students. A general set up is to relate geometric properties of a space with algebraic properties of its group of symmetries.

In this project, we explore the relation between the curvature of a space and the algebraic properties of its group of symmetries. Notions of curvature measure how far a space is from being flat. We will be working with two-dimensional spaces made of polygons and study combinatorial notions of curvature on them. For example, a tessellation of the Euclidean plane by squares will be considered flat, and a regular tessellation of the hyperbolic plane will have negative curvature. For concrete spaces with negative curvature, the student will identify algebraic properties of certain groups of symmetries by applying recent results in the literature.

The only background required is familiarity with the notion of group and interest in geometric ideas.

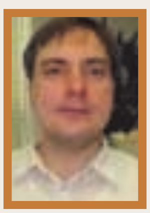
Dr. Eduardo Martinez-Pedroza
Assistant Professor in Mathematics
emartinezpedroza@mun.ca



Counting fine gradings on classical simple Lie algebras

If A is an algebra and G is a group then a *grading* by G on A is a vector space decomposition $A = \bigoplus_{g\in G} A_g$ such that $A_g A_h \subseteq A_{g\star h}$ where $\star$ is the operation in G. For example, take the algebra of 2 × 2 complex matrices $A = M_2(\mathbb{C})$ and write $A = A_{[0]} \oplus A_{[1]}$ where $A_{[0]}$ is spanned by the matrix units E_{11} and E_{22} and $A_{[1]}$ is spanned by E_{12} and E_{21}. Then we have $A_{[0]} A_{[0]} \subseteq A_{[0]}$, $A_{[0]} A_{[1]} \subseteq A_{[1]}$, $A_{[1]} A_{[0]} \subseteq A_{[1]}$ and $A_{[1]} A_{[1]} \subseteq A_{[0]}$, so the decomposition $A = A_{[0]} \oplus A_{[1]}$ is a grading by the additive group $\mathbb{Z}_2 = \{[0],[1]\}$. This grading can be *refined* as follows: take $A_0 = A_{[0]}$ and split $A_{[1]} = A_{-1} \oplus A_1$ where A_1 is spanned by E_{12} and A_{-1} is spanned by E_{21}. Then $A = A_{-1} \oplus A_0 \oplus A_1$ is a grading by the additive group $\mathbb{Z}$, with A_k defined to be zero for $k \notin \{-1,0,1\}$. This $\mathbb{Z}$-grading on $M_2(\mathbb{C})$ is *fine* in the sense that it cannot be further refined. It turns out that the number of fine gradings by abelian groups on the algebra $M_n(\mathbb{C})$ equals the number of abelian groups whose order is a divisor of n. For $n = 2$, there are two fine gradings: the above $\mathbb{Z}$-grading and the $(\mathbb{Z}_2 \times \mathbb{Z}_2)$-grading obtained by taking $A_{([0],[0])}$ spanned by the identity matrix and each of the remaining three components spanned by one of the Pauli matrices.

The algebra $M_n(\mathbb{C})$ contains three interesting subspaces: traceless matrices $\mathfrak{sl}_n(\mathbb{C})$, skew-symmetric matrices $\mathfrak{so}_n(\mathbb{C})$ and symplectic matrices $\mathfrak{sp}_n(\mathbb{C})$. Each of these subspaces is closed under the *commutator* operation $[a,b] = ab - ba$, so it is a *Lie algebra*. These Lie algebras are *simple* in the sense that they do not have nonzero proper ideals. We have a classification of all fine gradings on the Lie algebras $\mathfrak{sl}_n(\mathbb{C})$, $\mathfrak{so}_n(\mathbb{C})$ and $\mathfrak{sp}_n(\mathbb{C})$ in terms of orbits of certain actions of matrix groups over the field of two elements. The summer project would consist of exploring the combinatorics of these actions and counting the number of orbits (which is the same as the number of fine gradings) for various values of n.



Dr. Mikhail Kotchetov
Associate Professor in Mathematics
mikhail@mun.ca

Optimization of oil reservoir development

The viability of many projects related to the oil industry of Newfoundland and Labrador is sensitive to many operation parameters. Of particular interest is the question of where a set of injection/production wells should be placed to maximize profit over some time horizon into the future. It is not possible to experiment physically with the placement of wells. Instead, we will use oil reservoir simulators to computationally approximate production rates for oil and water and then evaluate an economic model of the value of production. Exhaustively evaluating the reservoir simulator for every possible configuration of wells would require millions of expensive computer simulations, well beyond the means of available resources. This project involves the development of fast strategies to search and efficiently identify the optimal configuration of wells. The student will learn about modern oil reservoir simulators, the mathematics behind them and the latest techniques in numerical optimization. Required skills: Matlab, some high level programming language, numerical methods, and partial differential equations.

Dr. Ronald Haynes
Associate Professor in Mathematics
rhaynes@mun.ca

Designing a multiscale computer model for petroleum reservoir simulation

The success of many projects related to offshore oil industry of Newfoundland and Labrador depends on improved understanding of the very complex fluid flow in geological reservoirs. A petroleum reservoir is a porous medium or rock formation that contains hydrocarbons. The study of petroleum reservoir simulation aims to find ways and means of optimizing the recovery of hydrocarbons. Since the primary recovery process leaves 70%-85% of hydrocarbons in the reservoir, an improved understanding of enhanced recovery methods remains an everlasting challenge to the field of petroleum reservoir engineering. In order to understand fully why hydrocarbons are trapped in rock formations and how to extract most of the trapped hydrocarbons, one must understand two principal issues. First, an understanding of the characteristics of the porous medium would tell us why hydrocarbons are trapped in the reservoirs. Second, the study of the complex characteristics of multiphase flow in a porous medium would enable the extraction of trapped fluid, thereby optimizing enhanced oil recovery techniques. According to current state-of-the-art theoretical and experimental knowledge, the extreme degree of complexity of fluid flow in reservoirs can be studied optimally using sophisticated computer models. On the other hand, physical models and observational data can be used to characterize geological models that describes a rock formation.

A goal of the proposed research is the development of a next generation multiscale computer model for studying the potential for enhanced oil recovery techniques as well as the storage of CO_2 in mature oil reservoirs. In this direction, I plan to extend an existing multi-scale computational fluid dynamics~(CFD) model to study multi-phase flow in porous media.

Dr. Jahrul Alam
Assistant Professor in Mathematics
alamj@mun.ca

Metastable quantum states: Master equation and resonance theory

Most bound states (stationary states) of a quantum system become unstable (decay slowly) when the system is coupled to another, large quantum system. This is due to irreversibility induced by the coupling. The student will learn two techniques for analyzing the dynamics of metastable (long-lived) quantum states: (1) the master equation method and (2) the quantum resonance method. The former is well established and widely applied in modern physics (e.g. quantum optics). However, in certain physically naturally emerging situations, predictions of the master equation approximation are accurate only on an intermediate time-scale ("weak coupling limit"). In particular, that approximation may predict the existence of many stationary states, even for systems known to have a unique such state. The reason for this is that up to a time-scale resolved by the master equation, many states look stationary, but beyond intermediate times they are in fact decaying. The quantum resonance method describes the dynamics accurately for all times. It can thus be used to separate the dynamics of "fake" stationary states (predicted by the master equation technique) from that of the true one. The goal is to (a) understand both techniques at a level permitting to carry out analyses of concrete models, and (b) to show directly the above mentioned scenario for a simple model.

Dr. Marco Merkli
Associate Professor in Mathematics
merkli@mun.ca

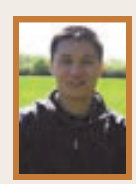
Geometry of quantum states

As the prospect of building quantum computers becomes more concrete, Quantum Information Theory is now one of the most active fields in science. Quantum entanglement (i.e., nonseparability) is thought to be at the heart of Quantum Information Theory. It is the key ingredient in modern quantum algorithms, such as Shor's algorithm for integer factorization and Deutsch-Jozsa algorithm.

One of the central problems in Quantum Information Theory is to understand the geometric and/or probabilistic aspects of subsets of quantum states, for instance, separable quantum states and entangled quantum states. Knowledge about the geometry of the subset of entangled quantum states is essential for identifying and quantifying quantum entanglement, and thus is crucial for designing and analyzing quantum algorithms.

In this project, we will study the following basic questions: (1) what is the relative size of entanglement (or other interesting properties)? (2) how to determine whether a quantum state is separable or entangled? Students will learn about quantum entanglement, criterion for detecting entanglement (or separability), measures on quantum states, etc. The required background is Linear Algebra, especially the theory of matrix.

Dr. Deping Ye
Assistant Professor in Mathematics
deping.ye@mun.ca



Combinatorial designs and structures

One of the several disciplines within combinatorial mathematics is that of design theory. Central to this field of study is the concept of a combinatorial design, which consists of a set *V* of elements called points together with a set *B* of subsets of *V* called blocks that conform to certain properties, such as requiring each block of *B* to have the same cardinality *k* as well as requiring that each 2-subset of *V* be found in the same number of blocks of *B*).

There are many open problems regarding designs, including questions such as whether designs with certain parameters exist, and if so, how many are there? Designs can also be associated with other structures, such as graphs in which the vertices are themselves blocks of a design and adjacency of vertices is based on the blocks having a specified number of points in common. While various properties of these graphs have already been discovered, there are several interesting questions that remain to be answered.

As an added bonus to a USRA in combinatorics at Memorial University, note that Memorial University will be hosting a four-week AARMS Summer School in 2012, during which guest lecturers will deliver two courses involving combinatorial mathematics. One of these will focus specifically on combinatorial designs and graph decompositions.

Dr. David Pike
Professor in Mathematics
dapike@mun.ca

The geometry of black holes

Black holes are among the most elementary and fascinating objects of study in mathematical physics. They arise as solutions of Einstein's theory of general relativity characterized by an event horizon, a region of space time from which nothing can escape. Further, they serve as a theoretical setting in which quantum effects due to intense gravitational fields can be studied. As a result, they are highly relevant from the point of view of constructing a consistent theory of quantum gravity.

Leading candidates for such a theory, such as string theory and the gauge theory/gravity correspondence (AdS/CFT) lead to extra spatial dimensions. Black holes in higher dimensions arise naturally within this framework and their study has led to a number of new insights, such as a microscopic derivation of Hawking's semi-classical entropy formula and holography.

An important problem is to classify black hole solutions of Einstein's equations in higher dimensions. However, the gravitational field has more degrees of freedom and there are a great number of possibilities. It has recently emerged that exotic, non-spherical black holes (doughnut-shaped 'black rings') are possible and that, in general, black holes are specified not only by their mass and spin, as is the case in ordinary four-dimensional space time. This project will focus on methods aimed at analyzing a certain class of black holes for which Einstein's equations can be separately solved 'near' the black hole event horizon itself. This will allow for a systematic treatment of the geometry and topology of these black holes. An overall aim is to place these results within the context of string theory and AdS/FT.

Although a first course in differential geometry or general relativity would be very helpful, it is also adequate to have a good grasp of techniques in mathematical physics: vector calculus/analysis, ordinary differential equations, and mechanics.

Dr. Hari Kunduri
Assistant Professor in Mathematics
hkkunduri@mun.ca

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